Climate Change and Uncertainty

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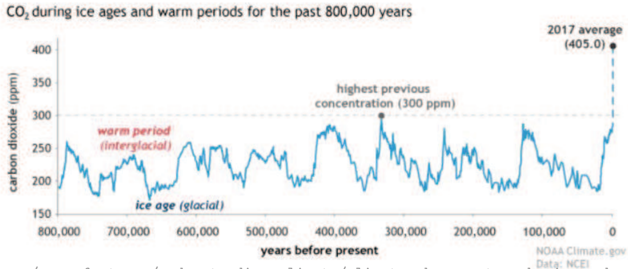
Drawing in particular on work with:
Svenn Jensen (OsloMet), Derek Lemoine (U Arizona),
Ben Crost (Urbana Champaign)

Motivation

Climate change is unprecedented.



www.climate.gov/news-features/understanding-climate/climate-change-atmospheric-carbon-dioxide

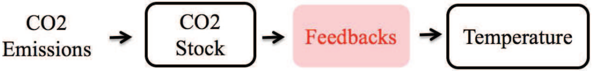
How to quantify optimal/reasonable/good climate policy?

- combine economic modeling, science, and “subjective judgements”
- use integrated assessment models (IAMs)
- This talk: How to capture “subjective judgements” alias uncertainty & what is the impact on the policy level?

Uncertainty Example: Feedback and Climate Sensitivity

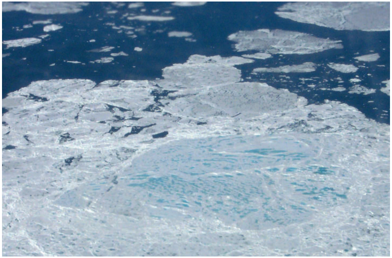
II. Feedbacks, Uncertainty, & SCC

What are we uncertain about? **Climatic feedbacks**



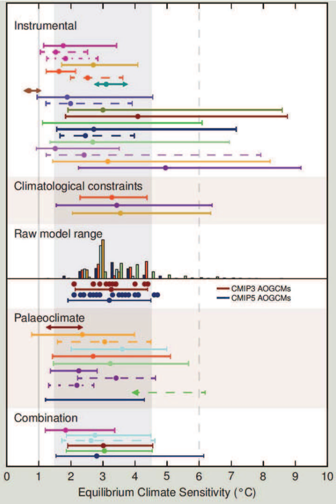
Examples of feedbacks:

- Water vapor,
- Melting permafrost,
- Lapse rate response
- Gulf stream weakening
- Ice-albedo . . .



Uncertainty Example: Feedback and Climate Sensitivity

What are we uncertain about? **Climate Sensitivity**



- Warming from doubling CO_2 concentration w.r.t. pre-industrial level (logarithmic relation)
- IPCC:
  - No best guess (anymore)
  - Within 1.5°C and 4.5°C with 2/3 *subjective* probability.
  - Much higher values possible.
- Very different worlds at 1.5°C, 4.5°C, or 6°C.

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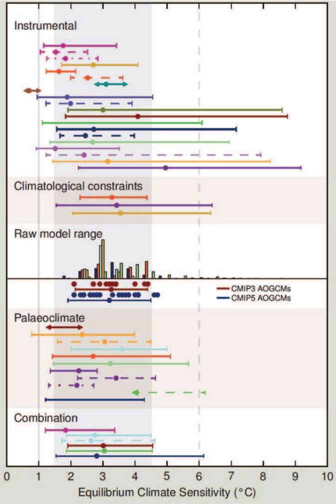

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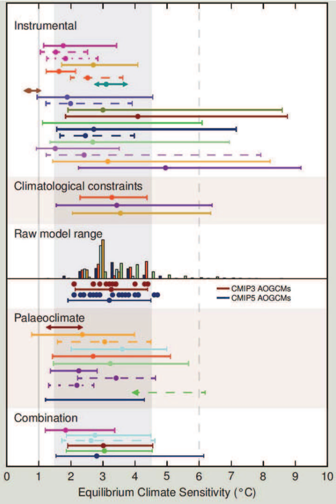

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Uncertainty Examples

Other uncertainty examples:

- Damage uncertainty
- Technological uncertainty (adaptation, mitigation)
- Can we outgrow climate change?
- Tipping points: (potentially irreversible) shifts in climate or ecosystem dynamics
- Policy effectiveness & uncertainty
- Strategic uncertainty (global game)

Methodological Overview

Methodological background:

- 2-3 decades of *deterministic* models: use best guess
- 1-2 decades of "Monte Carlo": average many deterministic worlds
- recent years + Kelly & Kolstad (1999): stochastic IAMs

Difficulty with the latter: "Curse of dimensionality"

Current approaches

- reduced form models
- DICE-based numeric models
- closed-form models
- general analytic insights
- advanced numeric approximation methods

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Questions

Questions guiding the presentation:

How does uncertainty affect

- the level of optimal* policy?

  *=advice on how we should set/change policy levels because of uncertainty
- what matters most for the result?
  - parametrically,
  - structurally,
  - conceptually

Roadmap

Will focus on example of climate (sensitivity) uncertainty

1. Intro
2. Numeric quantification – analytic structure
3. Role of learning
4. Tipping Points – numeric quantification – structure (does tipping change things?)
5. Closed-form analysis – parametric & structural insights
6. Conclusions

Related Literature

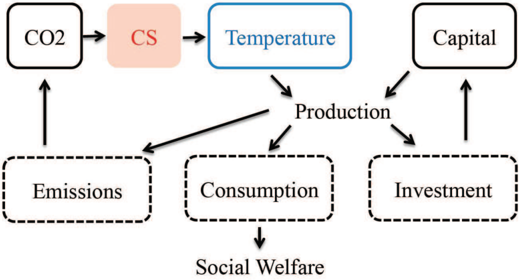
Some literature on Climate sensitivity uncertainty

- The numeric pioneers: Kelly & Kolstad (1999, JEDC)
- Other numeric work: Leach (2007, JEDC), Kelly & Tan (2015, JEEM), Lemoine and Rudnik (2018, WP),
- More stylized: Weitzman (2009 ReStat, 2012 ERE) & Millner (2013, JEEM) (fat tails), Dietz & Venmans (2019, JEEM)
- Partly analytic: Lemoine & Traeger (2014 AEJ:Policy, 16 JEBO), Golosov et al. (2014, E), Traeger (2014/18, WP ACE), Anderson et al. (2016, WP), Lemoine (2017, WP), Hambel & Kraft (2018, WP), Van den Bremer & van der Ploeg (2019, WP)

Similar but less literature on growth and damage uncertainty.
Next section based on *Pricing Climate Risk* with Svenn Jensen.

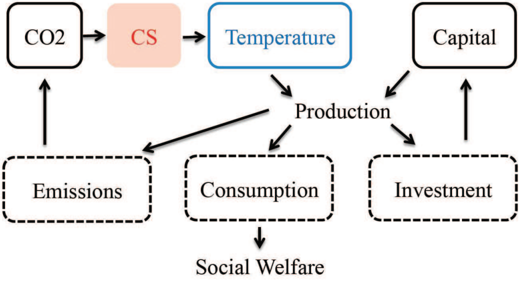
Social Cost of Carbon

DICE-style integrated assessment model



Social Cost of Carbon

What is the welfare cost of one additional ton of emissions E_0?



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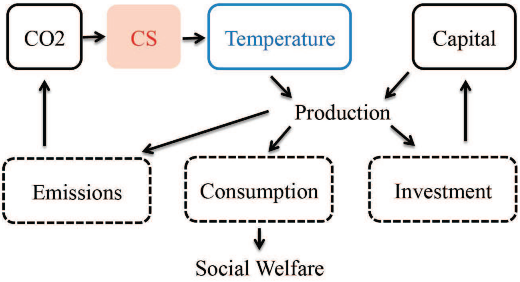

$$SCC_0 = -\frac{1}{u_0'(c_0)} \mathbf{E}_0 \sum_{t=1}^{\infty} \sum_{\tau=1}^{t} u_t'(c_t) \frac{\partial F_t}{\partial T_t} \frac{\partial T_t}{\partial CO_{2,\tau}} \frac{\partial CO_{2,\tau}}{\partial E_0}$$

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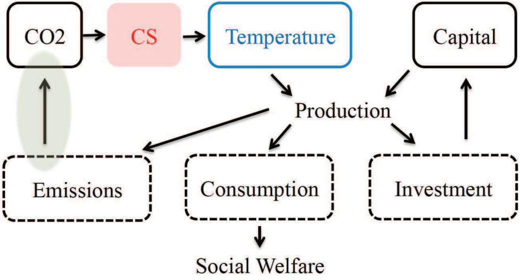

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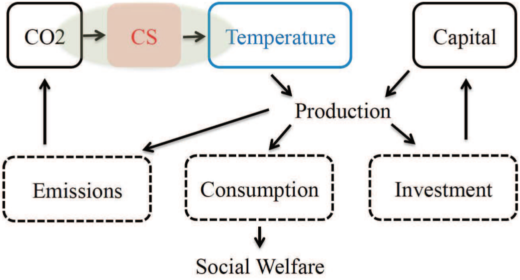

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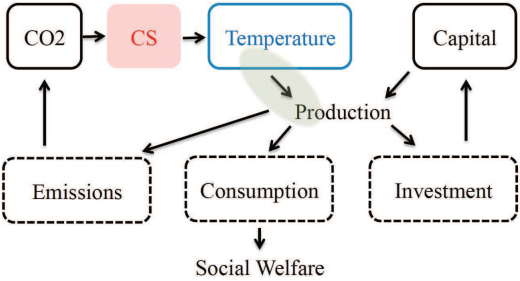

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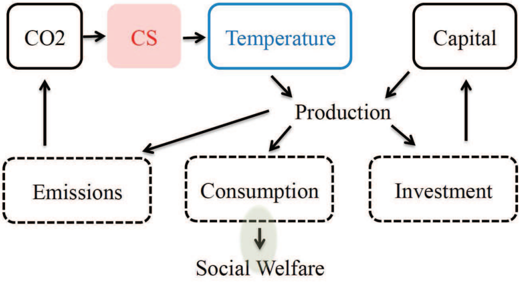

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Climate Risk Premium: Formula

When do we get a climate risk premium?

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Positive premium iff SCC convex in climate sensitivity s.

- Jensen's inequality

Sign not obvious, s affects:

1. temperature sensitivity to emissions $\frac{\partial T_t}{\partial CO_{2,\tau}}$ (linear),
2. temperature *level* T_t and thereby
   - productivity
   - consumption level c_t.
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Climate Risk Premium: Formula

Recall *precautionary savings* motive:

- Uncertainty increases my savings if
  more wealth reduces my pain from shocks (risk aversion)

Captured by

- Prudence: Prud$= -\frac{u'''}{u''} * c$

rather than

- Risk Aversion: RRA$= -\frac{u''}{u'} * c$

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and analogously define for damages:

- Dam$_2$$= \frac{F''}{F'} * T$: Damage convexity in temperature,
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Keep in mind:

- climate sensitivity hits various terms of the formula

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Climate Risk Premium: Formula

Also need to translate climate sensitivity uncertainty into resulting temperature and consumption uncertainty:

- $\epsilon_{T,s}$: Temperature elasticity w.r.t. climate sensitivity,
- $\epsilon_{c,s}$: Consumption elasticity w.r.t. climate sensitivity

Assumptions making dynamic problem analytically tractable:

A1 Fix Investment & emissions at deterministically optimal level

A2 Temperature is linear climate sensitivity
Basically by definition (satisfied in DICE)

$\epsilon_{T,s}$ will suffer most from assumption A1. But:
Can get a good approximation of true $\epsilon_{T,s}$ from a (numeric) deterministic model ("adjusted formula").

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Risk Premium

Proposition 1: Uncertainty over climate sensitivity increases the social cost of carbon contribution from a given period if and only if $X_t(\cdot) \equiv$

$$\underbrace{\mathrm{RRA}\,\epsilon_{c,s}}_{\substack{\text{direct}\\\text{risk}\\\text{aversion}}}\Big[2+\underbrace{\mathrm{Prud}\,\epsilon_{c,s}}_{\substack{\text{welfare}\\\text{prudence}}}+\underbrace{3\,\mathrm{Dam}_2\,\epsilon_{T,s}\Big]}_{\substack{\text{welfare}\\\text{economy}\\\text{interaction}}}+\underbrace{\mathrm{Dam}_2\,\epsilon_{T,s}\Big[2}_{\substack{\text{direct}\\\text{damage}\\\text{convexity}}}+\underbrace{\mathrm{Dam}_3\,\epsilon_{T,s}\Big]}_{\substack{\text{economy}\\\text{prudence}}}$$

is greater than zero.

Under a small risk approximation, the uncertainty premium is

$$\Delta SCC_0 \approx \sum_{t=1}^{\infty}\sum_{\tau=1}^{t}\underbrace{\beta^t\frac{u_t'(c_t)}{u_0'(c_0)}}_{\substack{\text{consumption}\\\text{discount}\\\text{factor}}}\underbrace{\frac{\partial F_t}{\partial T_t}\frac{\partial T_t}{\partial M_\tau}\frac{\partial M_\tau}{\partial E_0}}_{\substack{\text{marginal}\\\text{emission}\\\text{damage}}}\frac{\mathrm{Var}(s)}{2(\mathbf{E}s)^2}X_t(\cdot) \quad (1)$$

Temperature Stochasticity

Proposition 2: Under A1, A2, & small risk approximation: The temperature stochasticity premium is the sum of the per period contributions proportional to

$$\text{RRA}\,\epsilon_{c,T}\,[\text{Prud}\,\epsilon_{c,T} + 3\,\text{Dam}_2] + \text{Dam}_2\,\text{Dam}_3$$

Compare: Climate sensitivity uncertainty premium was

$$\text{RRA}\,\epsilon_{c,s}\,[2 + \text{Prud}\,\epsilon_{c,s} + 3\,\text{Dam}_2\,\epsilon_{T,s}] + \text{Dam}_2\,\epsilon_{T,s}\,[2 + \text{Dam}_3\,\epsilon_{T,s}]$$

$\hookrightarrow$ Stochasticity premium misses the direct risk aversion (small) and the direct damage convexity effect (large).

Reason: Climate sensitivity affects the temperature response to a given emission unit, a simple temperature shock does not.

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Temperature Stochasticity

Important implication:

Empirical literature usually estimates "climate change impact" from "temperature stochasticity".

Yet

Temperature stochasticity and climate change trigger different optimal responses. Extrapolation of the economic response to temperature shocks as compared to climate change is more complex and difficult than usually acknowledged.

Robustness

Quantification of risk premia (formula vs numeric stoch model)
in USD per ton of Carbon (USD/tC)

RRA=Arrow-Pratt risk aversion ; PRTP= time preference

Risk Premium in USD/tC	Formula Orig	Formula Adjusted	Full Model	error
RRA=2, $\rho = 1.5$, DICE13	19.0	15.7	15.8	0.7%
RRA = 1.45	29.5	21.1	21.4	1.5%
PRTP $\rho = 0.5$	34.0	22.3	23.0	3.1%
Mueller-Watson growth	15.7	13.8	14.6	5.5%
DICE 2007 Damages	15.8	13.9	13.0	6.7%
cubic Damages	70	42.3	48.6	12.9%

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For cubic small risk approximation kicks in (conjecture)

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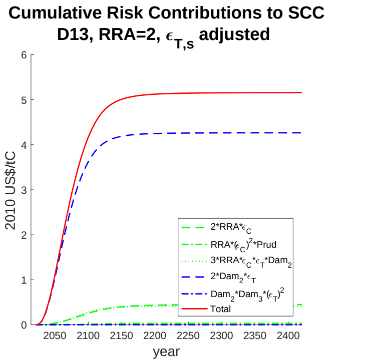
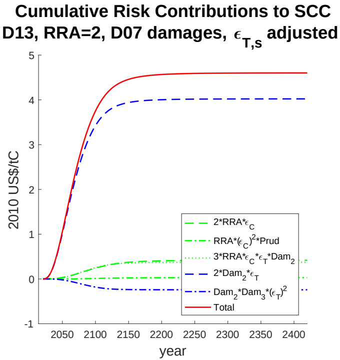
Climate Risk Premium - Characteristics

2. What are the important drivers? Adjusted $\epsilon_{T,s}$

Note: USD per unit of variance $(\sigma^2 \approx 3)$





$D(T) = 1 - 0.27T^2$ in % of output

$D(T) = 1 - \frac{1}{1+0.0028T^2}$ frac. of outp.

Climate Risk Premium - Characteristics

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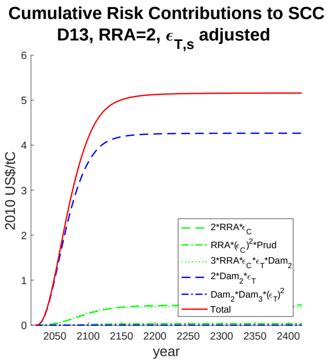

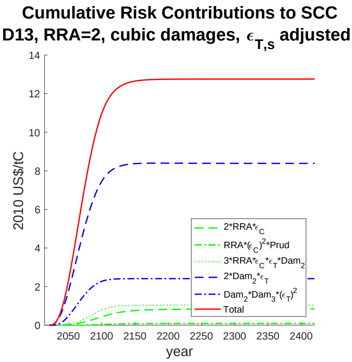

$D(T) = 1 - 0.27T^2$ in % of output

$D(T) = 1 - 0.1068T^3$ in % of output

Summary

Climate sensitivity risk premium

- 25-30% premium, about US$ 15-20 in base version of DICE
- Damage convexity most important contributor
- Risk aversion and third order damage curvature moderately relevant
- Prudence irrelevant
- climate sensitivity versus temperature stochasticity have qualitatively & quantitatively different impacts

Roadmap

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2. Numeric quantification – analytic structure
3. Role of learning
4. Tipping Points – numeric quantification – structure (does tipping change things?)
5. Closed-form analysis – parametric & structural insights
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Anticipated Learning

Same model with

Anticipated Bayesian Learning

- Future decision makers will have more information on climate dynamics than present decision makers.

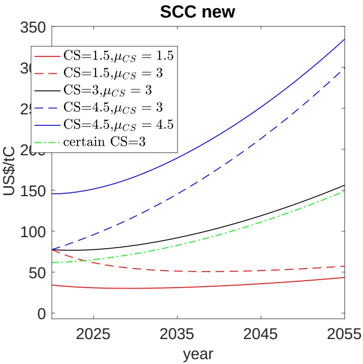
Should we wait and see?

Bayesian learning model

- hold a belief about climate sensitivity (prior)
- Global surface temperature subject to iid shocks
- infer “realized” climate sensitivity from temperature
- update belief about climate sensitivity

Anticipated Learning: Illustration

Learning under different true climate sensitivities

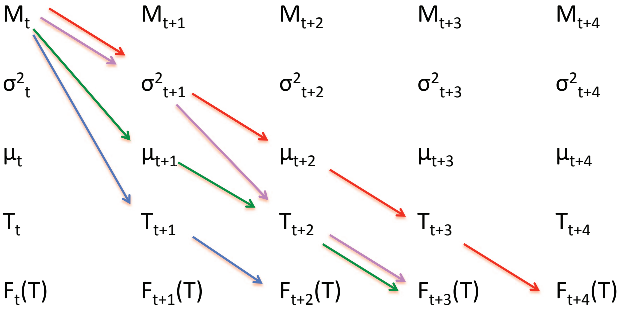


Prior is

- 3 Celsius in the present
- illustration of updating for "expected draws"

Anticipated Learning: “Analytic reasoning”

How does the anticipation of learning affect the SCC?



Anticipated Learning: SCC formula

How does the anticipation of learning affect the SCC?

Without learning:

$$SCC_0 = -\frac{1}{u_0'(c_0)} \mathbf{E}_0 \sum_{t=1}^{\infty} \sum_{\tau=1}^{t} u_t'(c_t) \frac{\partial F_t}{\partial T_t} \frac{\partial T_t}{\partial M_\tau} \frac{\partial M_\tau}{\partial E_0}$$

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With learning:

$$
\begin{aligned}
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&- \frac{1}{u_0'(c_0)} \mathbf{E}_0 \sum_{t=1}^{\infty} \sum_{\tau=1}^{t} \sum_{i=1}^{\tau} u_t'(c_t) \frac{\partial F_t}{\partial T_t} \frac{\partial T_t}{\partial \sigma_\tau^2} \frac{\partial \sigma_\tau^2}{\partial M_i} \frac{\partial M_i}{\partial E_0} \\
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\end{aligned}
$$

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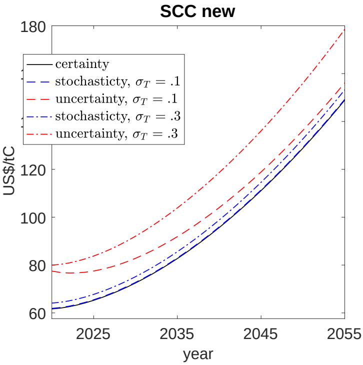
\end{aligned}
$$

Anticipated Learning: Result

How does the anticipation of learning affect the premium?



Dash to dash-dot:

- lower speed of learning
- by increasing stochasticity

$\hookrightarrow$ only a stochasticity effect

Conclude:

- Learning has little to no impact on **today's** SCC
- Don't "wait and see"

Roadmap

Will focus on example of climate (sensitivity) uncertainty

1. Intro
2. numeric quantification – analytic structure
3. Role of learning
4. Tipping Points – numeric quantification – structure (does tipping "change things"?)
5. Closed-form analysis – parametric & structural insights
6. Conclusions

Motivation: Economics & Media Attention

Tipping points are used as major argument for a 2°C target (adopted e.g. in Copenhagen Accord & Paris Agreement)

Hot air

The Economist

Are models that show the economic effects of climate change useless?

Oct 5th 2013 | From the print edition

Like 314 Tweet 272

MODELS simplify. They are supposed to. It is a feature, not a bug. Their formulae may be complex but models deliberately omit some things in order to focus on others. They also leave out factors that cannot be modelled satisfactorily. That is understandable, but what if the factors that cannot be modelled make a huge difference to the outcome? And what if the things that are excluded tend to produce a systematic bias in the results?

Oct 2013: "Are models that show the economic effects of climate change useless?", "climate models play down or leave out "tipping-point" risks that may not affect the climate yet but could do one day",

Our Contribution

IAMs with Tipping points: Lemoine & Traeger (2014,16,16); Cai & Lontzek (2016,19). Lemoine & Traeger (2016, NCC):

- incorporates three kinds of tipping points into a DICE-style integrated assessment model (IAM)
  - 1 affecting *CO_2 accumulation* in atmosphere CO_2 flow out of atmosphere drops by 50%
  - 1 affecting the *warming feedback* eql warming from 2xCO2 3°C to 5°C (climate sensitivity)
  - 1 directly affecting *economic damage* sensitivity (quadratic→cubic damages)
- analyzes interaction between different tipping possibilities
- derive optimal climate policy
- derive welfare cost of delaying optimal policy

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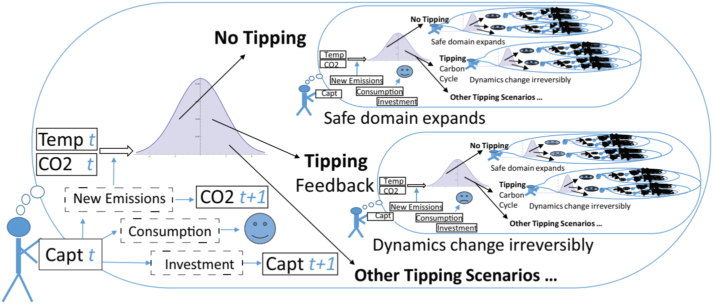
Our Contribution

Our approach

- optimize policy under
  - Bayesian uncertainty about the threshold location
  - anticipation of optimal future, state-dependent response to potential threshold crossing

Anticipates future observation, updated prior, optimal consumption and policy responses



Results: Interactions

Background DICE

- has “ad-hoc” damage adjustment for missing tipping points approximately doubles damage coefficient and SCC
- We take ad-hoc adjustment out and model explicitly

Our Result:

- The three stochastic tipping points double the SCC

↪ *back to original DICE for near term optimal tax*

(somewhat different dynamics also w/o actually tipping)

Which interactions are most relevant to SCC?

- feedback & damage: +35%
- carbon sink & damage: +22%
- carbon sink & feedback: +6%
- Triple interaction: +50%

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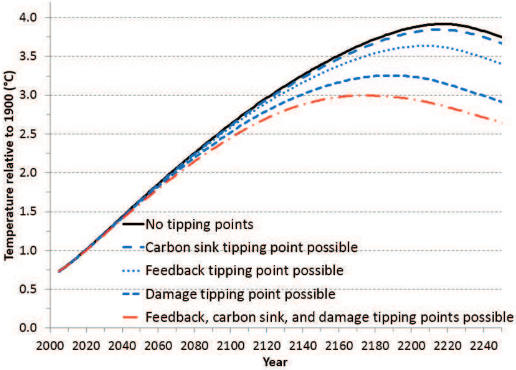
Results: Temperatures

Temperature along optimal trajectory conditional on not having tipped



Peak temperature decreases from almost 4°C to just below 3°C. Peak temperature reduction slightly less than adding individual TP effects.

Post-tipping response

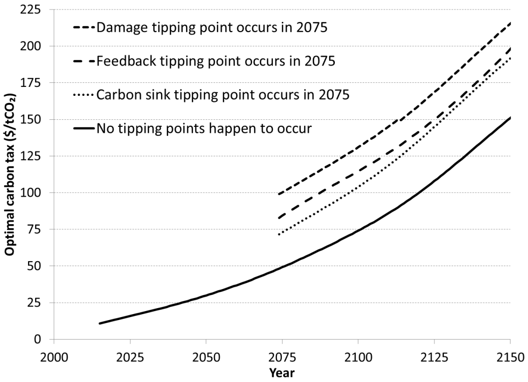


Figure: The effect of a tipping point on post-threshold policy. Simulations assume that all three tipping points are possible and that a particular one happens to occur in 2075 as the first tipping point.

Aversion and Ambiguity

Acknowledging probabilistic ignorance: Ambiguity Aversion

One tipping point only & original DICE damages:
Probability that threshold will ever be crossed

If tipping points not considered by the decision maker

68% chance of ever crossing the unknown threshold
(expected year of crossing conditional on eventual crossing: 2068)

	Tipping point			
	Climate sensitivity increased	Damage convexity increased	CO_2 sinks weakened	Non-CO_2 forcing increased
Uncertain threshold	0.46 (2050)	0.48 (2052)	0.61 (2062)	0.51 (2053)
Ambiguity aversion	0.45 (2049)	0.48 (2051)	0.61 (2062)	0.50 (2053)
Strong ambiguity aversion	0.40 (2046)	0.44 (2049)	0.61 (2061)	0.47 (2050)

(In parentheses: expected crossing year, conditional on crossing)

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Analytic Integrated Assessment Models (AIAMs)

Analytic IAMs ("of 2^{nd} generation"):
Starting point: Golosov et al. (2014)

- more insight than mere numeric models
- stochasticity: overcome "curse" of dimension
- strategic interaction: again overcoming "curse"

Here: ACE model (Analytic Climate Economy, Traeger 2018)

- obtains DICE-style realism in closed form
- decodes optimal carbon tax contributions
- explains & quantify uncertainty contributions

ACE in particular

- models **temperature** dynamics (global warming)
- solves with a **general risk attitude**
- **disentangles** RRA **from IES** (unity as in log utility)

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The ACE model

Structure of ACE:

- log-utility (deterministic)
- Cobb-Douglas production, using the additional
- Production function with generic energy sectors E_t:
  $Y_t = F(\boldsymbol{A}_t, \boldsymbol{N}_t, \boldsymbol{K}_t, \boldsymbol{E}_t)$
  with $F(\boldsymbol{A}_t, \boldsymbol{N}_t, \gamma\boldsymbol{K}_t, \boldsymbol{E}_t) = \gamma^\kappa F(\boldsymbol{A}_t, \boldsymbol{N}_t, \boldsymbol{K}_t, \boldsymbol{E}_t)\ \forall\gamma \in \mathbb{R}_+$.
- Resources, assumption: if scarce then essential
- Capital: Either Ramsey + simplified capital depreciation
  (10 year step AND/OR exogenous persistence correction)
  OR endogenous growth with dedicated "AK + energy"
  type capital sector (and arbitrary time step).

Solving for skip theorem

- feasible damage function & climate dynamics system

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Climate System

- Emissions:
  - Fossil fuel based energy sources $(1, ... I^d)$ emit: $\sum_{i=1}^{I^d} E_{i,t}$
  - Other (exogenous) CO_2 emissions: $E_t^{exogenous}$
  - Other (exogenous) non-CO_2 emissions: G_t
- Carbon cycle taken from DICE 2013:

$$\boldsymbol{M}_{t+1} = \boldsymbol{\Phi} \boldsymbol{M}_t + \boldsymbol{e}_1 \left(\sum_{i=1}^{I^d} E_{i,t} + E_t^{exogenous}\right) \tag{2}$$

- Radiative forcing (direct greenhouse effect of CO_2)

$$F_t = \eta \frac{\log \frac{M_{1,t}+G_t}{M_{pre}}}{\ln 2} \ . \tag{3}$$

  - Standard in numeric IAMs (& good Physics)
  - New to analytically tractable models

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Damages & Temperature Dynamics: Functional Forms

- Golosov et al. & others solve because *after suitable transformation* linear in states
- Linear-in-state models solved by affine value function

Proposition 1:

An affine value function of the form

$$V(k_t, \boldsymbol{\tau}_t, \boldsymbol{M}_t, \boldsymbol{R}_t, t) = \varphi_k k_t + \boldsymbol{\varphi}_M^\top \boldsymbol{M}_t + \boldsymbol{\varphi}_\tau^\top \boldsymbol{\tau}_t + \boldsymbol{\varphi}_{R,t}^\top \boldsymbol{R}_t + \varphi_t$$

solves ACE if

1. $k_t = \log K_t$, $\boldsymbol{\tau}_t$ is vector of $\tau_i = \exp(\xi_i T_i)$, $i \in \{1, ..., L\}$
2. Damages: $D(T_{1,t}) = 1 - \exp[-\xi_0 \exp[\xi_1 T_{1,t}] + \xi_0]$, $\xi_0 \in \mathbb{R}$,

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3. Temperature: $T_{i,t+1} = \frac{1}{\xi_i} \log\Big((1-\sigma_{i,i+1}-\sigma_{i,i-1})\exp[\xi_i T_{i,t}]$
   $+\sigma_{i,i+1}\exp[\xi_i w_i^{-1} T_{i-1,t}] + \sigma_{i,i-1}\exp[\xi_i w_{i+1} T_{i+1,t}]\Big)$,
   with weighting matrix $\boldsymbol{\sigma}$ capturing heat exchange
4. Parameters: $\xi_1 = \frac{\log 2}{s} \approx \frac{1}{4}$ and $\xi_{i+1} = w_i \xi_i = \frac{T_{eq}^{i-1}}{T_{eq}^{i}} \xi_i$.

Calibration

Summary of Proposition: Characterization of a class of IAMs with closed-form solution.

Calibration:

- Damage function close to DICE (initially slightly less convex, then more convex)
  - $\rightarrow$ damage parameter ξ_0 (semi-elasticity of output to exp temperature increase)
- Carbon cycle taken from DICE:
  - $\rightarrow$ Carbon transition matrix $\mathbf{\Phi}$
- Temperature dynamics calibrated to Magicc 6.0:
  - $\rightarrow$ "Heat" transition matrix $\boldsymbol{\sigma}$ and, in particular: speed of atmospheric temperature response to forcing σ^{forc}
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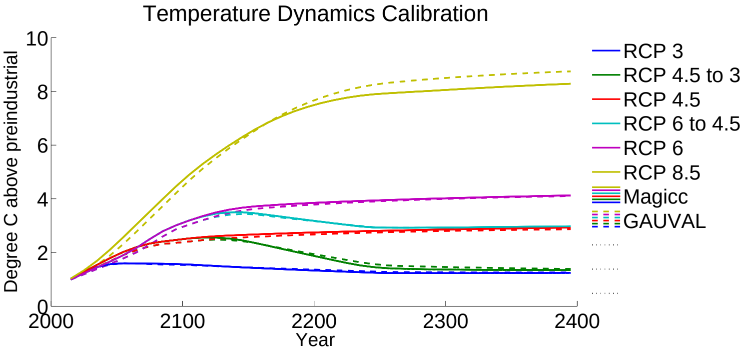
Climate - Temperature Dynamics

Modeling Atmosphere-Ocean Temperature dynamics

- A calibration to Magicc6.0 for IPCC's RCP scenarios,



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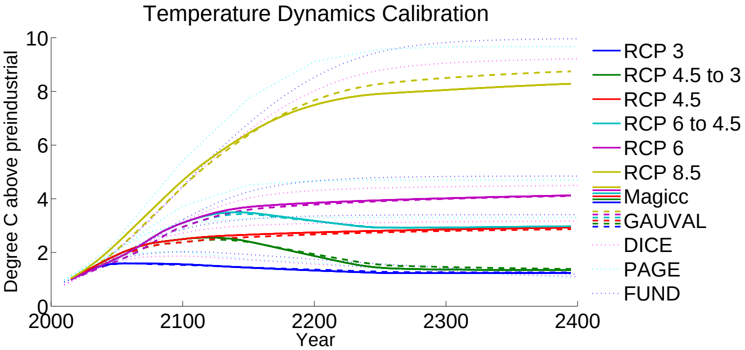

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The Social Cost of Carbon: Formula

The optimal carbon tax:

$$SCC_t = \frac{\beta Y_t}{M_{pre}} \underbrace{\xi_0}_{\text{damages}} \underbrace{\left[(\mathbf{1} - \beta\boldsymbol{\sigma})^{-1}\right]_{1,1} \sigma^{forc}}_{\text{climate dynamics}} \underbrace{\left[(\mathbf{1} - \beta\boldsymbol{\Phi})^{-1}\right]_{1,1}}_{\text{carbon dynamics}}$$

- discount factor β
- production Y_t
- preindustrial carbon M_{pre}
- damage parameter ξ_0 (semi-elasticity of net production)
- temperature dynamics $\boldsymbol{\sigma}$ ($\sim$ heat transfer) and, in particular:
- speed of atmospheric temperature response to forcing σ^{forc}
- carbon dynamics Φ (transition matrix)

The Social Cost of Carbon: Formula

The optimal carbon tax: (IMF 2018 USD, PPP, $\rho = 1.42\%$)

$$SCC_t = \underbrace{\frac{\beta Y_t}{M_{pre}} \underbrace{\xi_0}_{2.1\%}}_{11} \underbrace{\left[(\mathbf{1} - \beta \boldsymbol{\sigma})^{-1}\right]_{1,1}}_{1.1} \underbrace{\sigma^{forc}}_{0.54} \underbrace{\left[(\mathbf{1} - \beta \boldsymbol{\Phi})^{-1}\right]_{1,1}}_{4.3} = \mathbf{30}\ \frac{\$}{tCO_2}.$$

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A carbon cycle whose transition matrix $\mathbf{\Phi}$ satisfies **mass conservation** of carbon **implies a factor** $(1-\beta)^{-1} \approx \frac{\mathbf{1}}{\boldsymbol{\rho}}$ in the closed form solution of the **optimal carbon tax**.

The Social Cost of Carbon: Formula

The optimal carbon tax:

Drupp et al. (2018)'s expert survey $\rho = 0.5\%$ (median)

$$SCC_t = \underbrace{\frac{\beta Y_t}{M_{pre}} \underbrace{\xi_0}_{2.1\%}}_{\cancel{11}\, 12 \frac{\$}{tCO_2}} \underbrace{\left[(\mathbf{1} - \beta \boldsymbol{\sigma})^{-1}\right]_{1,1}}_{\cancel{1.1}\, 1.3} \underbrace{\sigma^{forc}}_{0.54} \underbrace{\left[(\mathbf{1} - \beta \boldsymbol{\Phi})^{-1}\right]_{1,1}}_{\cancel{4.3}\, 8.4} = \mathbf{\cancel{30}\, 72} \ \frac{\$}{tCO}$$

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At the pump: $\cancel{26}$ to 63 cents/gallon.
Note the factor 8 from carbon cycle.

A summary deterministic modeling

We can

- solve IAM with DICE-style realism (and maybe a bit more) in closed-form

My take of AIAMs under certainty:

- We can gain a lot of insights, including quantitative insights, about what matters how from the analytic formulas
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Uncertainty

Break with certainty equivalence.

Evaluating Uncertainty

1. Logarithmic utility is
   - Reasonable estimate for intertemporal substitution
   - Miserable estimate for risk aversion
2. Expected utility model is
   - unable to match high observed risk premia together with
   - low observed risk-free discount rate

Solution:

- Epstein-Zin-Weil preferences
  - IES=1 (logarithmic), deterministic tradeoffs
  - General CRRA risk attitude

details

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2. Expected utility model is
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   - low observed risk-free discount rate

Solution:

- Epstein-Zin-Weil preferences
  - IES=1 (logarithmic), deterministic tradeoffs
  - General CRRA risk attitude

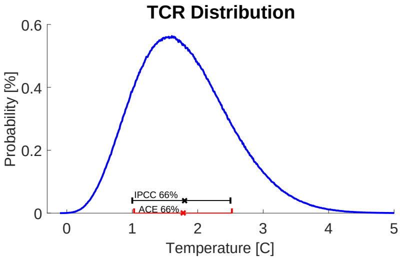
details

Uncertainty Models with closed-form solution

Example: Temperature uncertainty.
An autoregressive gamma process capturing uncertain response of temperature to atmospheric CO_2 concentrations.

(details)

Transient Climate Sensitivity (TCR)



matches IPCC's

- 66% probability that TCR in in $[1C, 2.5C]$
- expected 1.8C
- slight skew

Autoregressive Gamma and Temperature Uncertainty

Under such climate sensitity uncertainty
Optimal carbon tax changes from the deterministic SCC^{det} to

$$SCC^{unc} = SCC^{det} \underbrace{\left(1 + \underbrace{\frac{\bar{h}}{\sigma^{forc}}\theta_\tau(\cdot)}_{13\%\ (150\%)} + \underbrace{\frac{\bar{h}}{\sigma^{forc}}\epsilon(\cdot)}_{12\%\ (22\%)}\right)}_{\approx 1+25\% \approx 1+172\%}$$

where

- $\epsilon(\cdot)$: direct risk effect (even under risk neutrality)
- $\theta(\cdot)$: risk aversion (interacting with risk)

Similar numeric result to Jensen & Traeger (2013/19) discussed earlier and to Kelly & Tan (2015, JEEM)

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Time preference: Median response in Drupp et al.'s (2018,AEJ:Policy) expert elicitation:

Reduce pure rate of time preference from $\rho = 1.4\%$ to $\rho = 0.5\%$:

- SCC $\approx 200 \frac{USD}{tCO_2}$ (or 1.75 USD per gallon)
- reveals that the uncertain model is even more sensitive to time preference than the deterministic model.
- varying intrinsic risk aversion ($\alpha = 1$ in calibration): SCC $\approx 150 \frac{USD}{tCO_2}$ ($\alpha = 0.5$) / SCC $\approx 300 \frac{USD}{tCO_2}$ ($\alpha = 1.25$)

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Some Concluding Thoughts

- risk premia quantitatively important (still a lot to be done)
- std calibration: 25% premium for climate risk
- can be MUCH higher under more sophisticated calibration
- Deterministic SCC impact: carbon cycle >> temperature
- Uncertainty: clim sens uncert >> carb flow uncert
- discount rate even more important with fat-tails
- Structural drivers:
  - damage convexity more than standard risk aversion and prudence
  - disentangled=intrinsic aversion to risk matters a lot!
  - Tipping points or "smooth damages" somewhat similar
  - Ambiguous nature of uncertainty may not be as relevant under rationality assumption about dealing with ambiguity
- No support for wait and see because of uncertainty or learning

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